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Penelope Maddy, ***Naturalism in Mathematics.*** Oxford: Oxford University Press (1997), viii + 254 pp., $35.00 (cloth).

Let me start by saying that this is a good book, one that I highly recommend. It is interesting, original, and well-written, and it makes an important contribution to the philosophy of mathematics. It is also historically fascinating: it includes all sorts of interesting facts and insights about the history of mathematics and the natural sciences.

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The book is mainly about mathematical methodology, especially as it relates to undecidable propositions like the continuum hypothesis (CH). The central theses here are as follows.

(A) The CH question—i.e., the question of whether or not we should endorse CH—is a perfectly meaningful and legitimate question, worthy of mathematical pursuit.

(B) Philosophical considerations about the reality of abstract mathematical objects are wholly irrelevant to the argument for the legitimacy of the CH question; moreover, such considerations are also irrelevant to the question of whether or not we should endorse CH.

(C) Mathematical naturalism, i.e., the view that the theories and methodologies of mathematics do not stand in need of a supra-mathematical, first-philosophical justification.

(D) In light of (C), the proper way to argue for the legitimacy of the CH question is not to use philosophical arguments about mathematical realism and antirealism, but rather, to use *mathematical* arguments. Thus, a good naturalistic argument for the legitimacy of the CH question would proceed by (a) laying bare the nature of the goals, practices, and methodologies of mathematics; (b) explaining how these goals, practices, and methodologies are rational and justifiable (albeit justifiable from an internal, naturalistic point of view, as opposed to an external, first-philosophical point of view); and (c) explaining how, if we take these goals, practices, and methodologies for granted, it becomes clear that the CH question is a legitimate question, worthy of mathematical pursuit.

(E) The proper way to go about deciding whether or not we should endorse CH is to hunt for new set-theoretic axioms that might settle the question. Moreover, arguments for and against axiom candidates of this sort are (when cogent) purely mathematical, i.e., not philosophical. We can appreciate how these arguments proceed by simply looking at the history and practice of mathematics. In particular, arguments here can draw upon (i) the intuitiveness of the given axiom candidate; (ii) pragmatic considerations concerning the overall attractiveness of the resulting theory; and (iii) considerations involving various mathematical maxims and how well the given axiom candidate harmonizes with these maxims (and how well it aids us in our attempt to achieve various mathematical goals).

(F) Finally, it *could* turn out that mathematical arguments of the above sort do not settle the CH question. For there could turn out to be good mathematical reasons for embracing two different set theories that answered the CH question differently. But these reasons would not be philosophically based, and they would not show that the CH question was meaningless or illegitimate. They would be based on the mathematical attractiveness of the two theories and a belief that the benefits of pursuing both theories outweighed any desire to come up with a single, unifying set theory.

Maddy doesn't provide a detailed argument for all of the above points, but I think that this provides an accurate picture of the view that she is proposing. Her actual argument is centered around a case study that she makes of one proposition that has been taken as an axiom candidate, namely, the axiom of constructibility, i.e., 'V = L'. (Roughly, 'V = L' says that all sets in the cumulative hierarchy are definable by first-order formulae.) Maddy explains how there is a strong mathematical case against 'V = L' based on the fact that it is *restrictive* (because it rules out the study of hierarchies containing non-constructible, i.e., non-definable, sets). She explains that to endorse a restrictive axiom of this sort would be irrational and unacceptable because it would fly in the face of one of the most fundamental goals of mathematical practice, namely, the goal of using set theory to provide a foundation for mathematics without encumbering it. This goal, as well as the goal of studying whatever structures are mathematically interesting, leads to a methodological maxim that Maddy calls MAXIMIZE, which dictates that set theory "should be as generous . . . as possible . . . [and] provide models for all mathematical objects and instantiations for all mathematical structures . . . [and the widest possible] range of available isomorphism types" (210–211). Thus,

in a nutshell, the naturalistic argument against 'V = L' is that (a) it violates MAXIMIZE, and (b) this is unacceptable because MAXIMIZE is an important (and internally justifiable) maxim of mathematics.

Maddy's claim that the debate about mathematical realism is irrelevant to the CH question—thesis (B) above—might seem a bit surprising. For, prima facie, it seems that the realism debate should be relevant here. In particular, it seems that (a) if there is a real universe of sets, then CH is either true or false of that universe, and (b) if there are no sets for CH to be true or false *about*, then it couldn't have a determinate truth value and, hence, there couldn't be a legitimate question here to answer. Now, Maddy argues that this traditional stance flies in the face of practice: regardless of whether realism or antirealism is true, there could turn out to be good mathematical arguments for CH, or ~CH, or the claim that there is no "right answer" here. But she doesn't say *how this could be*, or in other words, where the traditional stance goes wrong. My own view is that (a) Maddy is right here and (b) we can explain "how this could be" by rejecting traditional versions of realism and antirealism and developing and motivating alternative versions of these views that account for the fact that there could be good reasons for accepting CH or ~CH or neither (and that entail that mathematicians can and should decide what to say here without concern for the realism debate, i.e., by considering only mathematical arguments; see my 1998 for the details). I suspect that Maddy would approve of this general approach, for she acknowledges that some versions of realism and antirealism might be naturalistically kosher, i.e., consistent with mathematical practice.

The question of the naturalistic acceptability of mathematical realism brings us to an interesting facet of Maddy's book: one of its central theses is that the kind of realism that Maddy defended earlier in her career (1990) is *not* naturalistically kosher. Maddy argues, for instance, that her early view cannot account for the fact that the standard (and proper) argument against 'V = L' pays no attention to the "true nature of the actual universe of sets"—i.e., ignores the question 'Do there really exist any non-constructible sets in the cumulative hierarchy?'—and concentrates instead on the fact that 'V = L' is restrictive. More generally, Maddy argues that early-Maddian realism is inconsistent with MAXIMIZE, i.e., with the fact that mathematicians want to posit and study as many objects as they can, without concern for the question, 'Which of these objects really exist?' (But again, Maddy is not arguing against *all* versions of realism here. She allows that other versions of realism might be consistent with mathematical practice, and more specifically, with MAXIMIZE and the standard argument against 'V = L'.)[1] *Mark Balaguer, California State University, Los Angeles.*

REFERENCES

Balaguer, M. (1998), *Platonism and Anti-Platonism in Mathematics.* New York: Oxford University Press.

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1. I would like to thank Penelope Maddy for commenting on an earlier version of this review.

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